



1. (8 pts) An 800 gallon tank is filled with brine which contains 300 pounds of salt. Every minute two gallons of dilute brine containing .1 pound of salt per gallon are pumped in and two gallons of the well-mixed brine are pumped out. Let  $S(t)$  be the amount of salt in the tank (in pounds) at time  $t$ , where  $t$  is measured in minutes. Write down an initial value problem for  $S(t)$ ; show the units make sense.
2. (20 pts) A security camera is centered 50 feet above a 100-foot long hallway and can rotate, as in the figure above. What is the maximum rate of rotation of the camera required to track objects moving along the hallway at 6 feet per second? (Hint: find a formula for the rotation rate as a function of  $\theta$ .)
3. (15 Points) Consider a differentiable function  $F(x)$ .

(a) (4 points) What is the slope of the secant line through  $F(x)$  and  $F(3)$ ?

(b) (2 points) Write down the limit definition for  $F'(3)$ .

(c) (1 point) Suppose  $F(x) = \int_3^x e^{\arctan(t)} dt$ . What is  $F(3)$ ?

(d) (4 points) Given your answers to (a), (b), and/or (c), find

$$\lim_{x \rightarrow 3} \left( \frac{\int_3^x e^{\arctan(t)} dt}{x - 3} \right).$$

(e) (4 points) Find

$$\frac{d}{dx} \int_3^{1/x} e^{\arctan(t)} dt.$$

(Hint: Express the above in terms of  $F(x)$ .)

4. Let  $g(x) = \int_1^x \sqrt{1+t^2} dt$ .

• What is  $g'(x)$ ?

• What is  $g(x^3)$ ?

• What is  $\frac{d}{dx}g(x^3)$ ?

- (a)  $\sqrt{1+x^6}$
- (b)  $3x^2\sqrt{1+x^2}$
- (c)  $x^3(1+x^6)^{-1/2}$
- (d)  $3x^2\sqrt{1+x^6}$